On correcting Observations made with Equatorial Instruments. By T. R. Robinson, B.D. M.R.I.A. *Professor of Astronomy at Armagh.*

Read January 10, 1825.

ON being appointed to the Observatory at Armagh, the principal instrument of which is a magnificent Equatorial by Troughton, I was anxious to collect all possible information connected with the mode of using it. But in this I had little success; astronomers have investigated the theory of the Transit and Vertical Circle thoroughly, and almost every treatise on the science contains a variety of formulæ to facilitate *their* employment, while the essay of Sir G. Shuckburgh, and a chapter of Professor Vince's Pract. Ast. were my sole resources. My predecessors *here* seem to have felt the same difficulty; at least they have recorded no observations made off the meridian: I am even informed that it was in contemplation to mount the Equatorial in a vertical position, but this would certainly be unjust to its admirable maker, who I think never displayed greater ability than in its construction. At first sight nothing can be more facinating than the apparent facility which an Equatorial offers to the astronomer; the determining a star's place by an immediate reference to the pole instead of the zenith pleases by its simplicity; far more numerous observations can be taken of the same star in each day, and the power of observing with accuracy off the meridian, is of inestimable advantage in this variable climate. The great service which a telescope, provided with a micrometer and equatorial stand, can render astronomy is well known; and it may

shew how much more advantageous a machine must be which, with little less steadiness than a transit instrument, is capable of identifying the minute stars which are sometimes used as objects of comparison, to the accuracy of a few seconds. It is true that from the oblique and variable stress to which the polar axis of such an instrument is exposed, and the unequal effect of temperature on its supports, it cannot have the precision of a large Vertical Circle; but leaving to such the almost differential researches connected with parallax and aberration, much remains to be filled up, before we can say that we are acquainted with the Heavens. To accomplish this, with the assistance of a standard catalogue, the machines which we are considering are fully adequate; and it is surprising when we consider how many of them are in the hands of amateurs that so few results have been obtained by their means: except the Armagh instrument, and that which Mr. South uses with such zeal and ability, none that I know of seem to have contributed any addition to Science. But it must be owned that their use presents certain difficulties; the calculation of refraction and parallax is considerably involved, and these corrections affect both declination and right ascension; the adjustments are more numerous and complex than in vertical instruments, and much more easily disturbed; and the absolute impossibility of using a plumb-line for the verifications is unfavorable to accuracy. The refraction apparatus of Ramsden is commonly annexed to them, but it cannot be employed with any regard to the permanence of the collimation. Some of these inconveniencies I think I have been able to obviate; and in offering the formulæ which I use to the notice of the Academy, if I can induce any person who possesses an equatorial, to take it from its case, and avail himself of its powers, I shall be well rewarded:

jestic edifice, once the proud seat of imperial grandeur, after a lapse of time and change of circumstances, deserted, decayed and doomed to shelter the humble peasant or the shepherd's care. The sublime and lofty halls, pinnacles and towers, splendid but melancholy monuments of former magnificence, remain to exercise the talents of antiquarian learning and excite the admiration of ages. Such at the present day, is the language of Ireland. But after braving a thousand storms, it yet remains unimpaired and so will continue, *monumentum ære perennius.** This venerable fabric is the subject of our present consideration. We shall now proceed to consider its various parts, and in doing so, it will appear how far we are qualified for the attempt.

The Irish language, as before observed, contains within it the radices of the ancient Celtic. The affinities between the latter and the dialects derived from it can be better traced in the Irish than in any of the other existing branches of that great stock. The knowledge of it alone would yield more materials for a system of Etymology than any other language, the Hebrew excepted, and even more than we can be supplied with by all the laborious researches of etymologists put together, notwithstanding their having paid the most unremitting attention to the subject. Several classes of words in most of the Oriental languages bear visible marks of being derived from the same common parent as the Irish. Many of the northern languages have originated from the same source. If etymologies were

* It is now ascertained that the Irish language is spoken in the interior of many of the West Indian islands, in some of which it may be said to be almost vernacular. This curious fact is satisfactorily explained by documents in the possession of my respected friend, James Hardiman, Esq., author of the History of Galway. After the reduction of Ireland by Cromwell and his myrmidons, the thousands who were " shipped to the Carribbes" so these islands, were then called, " and sold as slaves," carried with them their language. *That* they preserved, and *there* it remains to this day.

$$\sin. \Pi = \frac{\sin. S. \sin. u}{\sin.(D + \Delta)}$$

substituting for sin. Δ, sin. u. cos. S; and developing

$$\sin. \Pi = \frac{\sin. S}{\sin. D} . \sin. u - \frac{\frac{1}{2} \sin. 2\,S}{\sin. D. \text{tang.} D} \sin.^2 u + \&c.$$

I have given the second terms of these expressions to estimate the accuracy of the first; but the greatest of them does not exceed 0″. 3 at 85°. *ZD*, even in the moon's parallax, and may therefore be neglected, and the equations become

$$\sin \Delta = \sin. u. \cos. S \qquad (1)$$

$$\sin. \Pi = \frac{\sin. u. \sin S.}{\sin. D.} \qquad (2)$$

If we put Brinkley's refraction for u, we evidently obtain R and ϱ, the refractions in *PD* and *AR*: it is of the form

$$u = m. \text{tang}. z. - \varkappa. \text{tang}. z.$$

The coefficient $\varkappa$ being found by dividing the numbers of Brinkley's second table by tang. z, as I have done in my copy of them; therefore

$$R = (m - \varkappa) \text{ tang}. z. \cos. S.$$

but putting $PE = \zeta$; (it is the common auxiliary arc used to find the *Z. D* from the Polar distance and hour angle) we have

$$\cos. S. \text{tang}. z. = \text{tang}. (D - \zeta)$$

therefore

$$R = (m - \varkappa). \text{tang}. (D - \zeta) \qquad (3)$$

or where the trouble of forming the tables of $\varkappa$ is not taken

$$R. = m. \text{tang}. (D - \zeta)\, c. \cos. S.$$

c being the number found in table II. When ζ is known, the calculation of the refraction in *P. D.* is little more difficult than that in altitude. This arc is so useful that I computed a table of its values for every minute in time of the angle P, up to 6^h. which was easily

done as the second differences are constant, but which of course varies with the latitude.

EXAMPLE.

Aldebaran was observed Jan. 15, 1824, $5^h.\ 20'.\ 20''$ off the meridian Bar. 30. 3. Ther. 40°.

Appar. N. P. D. $=$	$73°.\ 48'.\ 50''$	$m =$	6. 0345
$\zeta =$	$7.\ 1.\ 26$	$\varkappa =$	6282
$D-\zeta=$	$66.\ 47\ 24$	$m-\varkappa=$	5. 4063
Number T. 1. $=$	0. 2992	log. $(m-\varkappa)$	1. 73291
Log. Bar.	1. 48144	tang. $(D-\zeta)$	10. 36771
	1. 78064. Log. m.		2.10062
		Refract $=$	$126''.\ 07$

In taking out $\varkappa$, it is necessary to know *Z. D.* to a few minutes, and the quantities necessary for finding it, are already computed; but for those who do not like calculation, a celestial globe will give it near enough. Within two hours of the meridian it is given by this approximation:

$$\varkappa + D-\zeta + \frac{\text{tang.}\ a}{\text{tang.}(D-\zeta)}\ \text{sin.}\ \delta.$$

where $a =$ comp. latitude and $\delta = a - \zeta$.

The cosine of the angle of position also,

$$\text{cos.}\ S = 1 - \frac{\text{tang.}\ a}{\text{tang.}\ (D-\zeta)}\ \text{sin.}\ \delta.$$

The Refraction in *P. D* is negative when D is less than ζ, and this latter changes sign when the hour angle is greater than 90°.

The refraction in *AR* is by (2) in space,

$$\varrho = u\frac{\text{sin.}\ S}{\text{sin.}\ D} = \frac{u\ \text{cos.}\ S.\ \text{tang.}\ S}{\text{sin.}\ D.}$$

$$= \frac{R.\ \text{tang.}\ S}{\text{sin.}\ D.} = \frac{R.\ \text{tang.}\ P.\ \text{sin.}\ \zeta}{\text{sin.}\ D.\ \text{sin.}\ (D-\zeta)} \qquad (4)$$

which is negative when the star is west of the meridian if applied to the *AR*.

My instrument has 3 horary wires; and in observations off the meridian it is evident that the *Z D.* of a star must be different as it passes each of them; the refractions must also differ, and it may be doubted whether the formula just given, which belongs to the middle wire, represents the mean of the three. Let the interval of the wires in space = *I*, sin $S \times I$ = variation of z from one wire to another: developing tang. $(z+v)$ in terms of v.

$$\text{tang.}\,(z+v) = \text{tang.}\,z + \frac{v}{\cos.^2 z} + \frac{\sin.\,z}{\cos.^3 z}\,v^2 + \&c.$$
$$= \text{tang.}\,z \left\{ 1 + \frac{\sin.\,S}{\sin.\,z \cos.\,z}\,I + \frac{\sin.^2 S}{\cos.^2 z}\,I^2 + \&c. \right\}$$

but the refraction in *AR* at the middle wire

$$\varrho = \frac{(m-\varkappa)\,\sin.\,S.\,\text{tang.}\,z}{\sin.\,D.}$$

and that at the lower

$$\varrho' = \frac{(m-\varkappa)\,\sin.\,S.\,\text{tang.}\,(z+v)}{\sin.\,D.}$$
$$= \frac{m-\varkappa.\,\sin.\,S\,\text{tang.}\,z}{\sin.\,D.} \left\{ 1 + \frac{\sin.\,S.}{\sin.\,z\,\cos.\,z}\,I + \frac{\sin.^2 S}{\cos.^2 z}\,I^2 + \&c. \right\}$$
$$\varrho' = \varrho \left\{ 1 + \frac{\sin.\,a.\,\sin.\,P}{\sin.\,z\,\sin.\,2z}\,2\,I + \frac{\sin.^2 a\,\sin.^2 P}{\sin.^2 2z}\,I^2 + \&c. \right\}$$

For the upper wire, *I* changes its sign and we have for the mean of the three refractions

$$\tfrac{1}{3}\,(\varrho + \varrho' + \varrho'') = \varrho \left\{ 1 + \frac{8\,I^2}{3} \cdot \frac{\sin.^2 a.\,\sin.^2 P}{\sin.^2 2z} \right\}$$

the second member of which when $I = 6'$ does not exceed $0''.1$ at any altitude where observations are useful.

The formulæ (1) and (2), also give the corrections for parallax; let h be the horizontal parallax, p that in *P D*; ϖ that in *AR*

$$\sin. h' = \sin. h \sin. z.$$

$$\sin. p = \sin. h \sin. z \cos. S.$$

$$= \frac{\sin. h. \cos. a \sin. (D - \zeta)}{\cos. \zeta} \qquad (5)$$

Also,

$$\sin. \pi = \frac{\sin. h. \sin. z. \sin. S.}{\sin. D.}$$

$$= \frac{\sin. h. \sin. a. \sin. P.}{\sin. D.} \qquad (6)$$

There is yet a further use of the arc ζ: in observing polar distances with the equatorial, after reading off, the inclination of the polar axis must be ascertained by the declination level. This is seldom found correct, and the difference between it and the supplement of latitude must be applied as Index error. In general the level has been used only on the meridian, but this is obviously inaccurate; for observations made in other hour circles are affected by any irregularity in the pivots of the polar axis or any flexure which it may sustain. The framing of the Armagh Instrument (a plate of which is given in Dr. Rees's Cyclopædia) unites stiffness and lightness in the highest degree, yet it bends a little; which becomes apparent in observation from the *PD* microscopes being supported on the Equator instead of points 90° from it. Theory gives me that this flexure varies as the cosine of *P*, but as one of the cones which compose the polar axis is not as much drawn by its screws as the other three (which I find by heating each separately, while a lucid point is bisected by the *PD* wire) the origin of the angle is thrown to the westward, and the correction to be applied to the reading is

$$= - 11'' \times \cos. (P^\circ - 30^\circ.)$$

This of course is eliminated by reversed observations, and in other cases may be computed; but the axis-error is more satisfactorily determined by observing the level on the same hour circle

with the star: its polar distance should be the arc PO (see the figure) but

$$\text{tang. } PO = -\text{cotang. } PE$$
$$PO = 90^{\circ} + \zeta$$

therefore it is known, and any error which affects the instrumental determination of the star's PD, affects this arc equally.

Lastly it is necessary to estimate the errors which arise from the declination axis not being perpendicular to the polar axis, and from the latter not being parallel to that of the earth.

In the first case the observed PD is the side of the right angled triangle whose hypothenuse is the true, and calling the error of the axis n, we obtain the ordinary reduction,

$$\text{sin. } \Delta' = \frac{2 \text{ sin.}^2 \frac{n}{2}}{\text{tang. } D}$$

$$\text{sin. } \Pi' = \frac{\text{sin. } n}{\text{tang. } D.}$$

But this error is easily corrected, and the adjustment is not liable to vary.

The error of the polar axis is much more unmanageable; it changes with every variation of temperature, and the instrument can scarcely be touched without affecting it. Suppose its upper extremity raised e seconds, and moved westward m seconds; to find the values of Δ and Π, thence arising, we have

$$\text{cos. } D = \text{cos. } z. \text{ cos. } a + \text{sin } z. \text{ sin. } a \times \text{cos. Azimuth},$$

differentiating on the hypothesis of z constant,

$$\text{sin. } D. \, d\,D = \left\{ \begin{array}{c} d\,a \times (\text{cos. } z \text{ sin. } a - \text{sin. } z \text{ cos. } a. \text{ cos.} A.) \\ + d\,A. \text{ sin. } z. \text{ sin. } a. \text{ sin. } A. \end{array} \right\}$$

Substituting for $d\,a$, $d\,D$, and $d\,A$, their equivalents, e, Δ and $\frac{m}{\text{sin.} a}$, and for the coefficient of $d\,a$ its value cos. $P \times$ sin. D, we derive

$$\Delta = e. \text{ cos. } P + m. \text{ sin. } P. \qquad (7)$$

To find Π, differentiate, on the same hypothesis,

$$\cos. z. = \cos. a. \cos. D + \sin. a. \sin D. \cos. P.$$

and substituting for Δ its value just obtained we deduce

$$\Pi = -\frac{e. \sin. P}{\text{tang. } D} - \frac{m \sin. P}{\text{tang. } S \sin. D.} \qquad (8)$$

or where S is not known

$$\Pi = -\frac{e \sin. P + m. \cos. P -}{\text{tang. } D.} \frac{m}{\text{tang. } a}$$

or,

$$\Pi = -\frac{e \sin. P}{\text{tang. } D} - \frac{m. \sin. (D - \zeta) \cos. P.}{\sin. D \sin. \zeta}$$

When this correction is applied to the *AR* instead of the hour angle, its sign must be changed.

We obtain e by the declaration level, m by observing the distance of the meridian mark from the apparent meridian $= \Pi'$ for

$$m = -\tfrac{1}{2} \Pi' \times \sin. 2 a.$$

By the use of these corrections, I can venture to say, that the Armagh Equatorial gives results which are not unworthy of the present improved state of astronomy. The only error which remains is produced by slight variations of the supports of the declination level, connected with the extensive movements to which it is exposed. This however is scarcely ever 5″, and I hope to be able to counteract it. Should I succeed, I shall from time to time lay such of my results as are likely to be useful before the Academy.

T. R. ROBINSON.

Armagh, January 4, 1825.